

Stochastic Modelling of Blockchain Systems

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URPP Social Networks



Universität
Zürich ^{UZH}

- ① Introduction
 - Bitcoin network
 - Research Questions
 - Stylised facts
- ② Modelling approach
- ③ Results
- ④ Conclusions

The first slide should be generic about bitcoin. pervasiveness, impact, development.

Please insert some figure. See example.tex to see how to do it.

Networks in Bitcoin

In Bitcoin there are two networks at play: Economic transactions, P2P Network

The first slide should be generic about lithium, potassium, sodium, development.
Please insert some figure. The example on the next slide to do it.

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This is a note page

- Please, write here some notes of the general idea of what this slide contains

The first slide should be generic about bitcoin. pervasiveness, impact, development.

Please insert some figure. See example.tex to see how to do it.

Networks in Bitcoin

In Bitcoin there are two networks at play: Economic transactions, P2P Network

The first slide should be generic about lithium, potassium, sodium development.
Please insert some figure. See example on the next slide to do it.
In 30 seconds there are two networks at play: Economic transaction, P2P Network.

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Networks in Bitcoin

In Bitcoin there are two networks at play: Economic transactions, **P2P Network**

The first slide should be generic about lithium, potassium, sodium development.
Please insert some figure. See example on the next slide to do it.
In Slide 3 there are two networks of play. Elements: transition, VOP Network.

This is a note page

- Please, write here some notes of the general idea of what this slide contains

Specific on bitcoin network

- Here we are interested in the bitcoin P2P network.
DESCRIBE IT
- Consensus Protocol DESCRIBE IT
-

Please insert some figure. See `example.tex` to see how to do it.
If necessary, split this slide into many

- Note we are interested in the Bitcoin P2P network.

- Distributed IT

- Consensus Protocol DISCUSS IT

-

Please read entire figure. The message(s) we see here is/are the only one(s) if necessary split this slide into many

For this slide, the notes are very important.

- Give some examples about systems that can be described in terms of a Gillespie algorithm (without mentioning it explicitly)
- Now the match should be clear

- Can we model about systems that can be described in terms of a Gibbs algebra (without extending to quantum)?
- How far reach should be clear

Notes on what you meant

- Generic research question: Why modelling?
- And this is the second on Why modelling
- specifically we address in this talk
- What happens if we alter the nature of the Bitcoin topology in terms with efficiency

- Given research question: Why studying?
- And this is the second or Why studying?
- Specifically our interest in this talk
- What happens if we alter the nature of the Bloom topology in terms with efficiency

Notes on what you meant

- The P2P bitcoin network : size, average degree, cite references, mention some things that are known
- The distribution of hashing powers
- Some other thing you may fancy adding

- The GDP growth rate is not average degree, the reference, neither some things that are known
- The distribution of trading partners
- Some other things you may have added

Notes on what you meant

Model ingredient

- Network Topology.
Network topology provides the connection relationship among nodes in consensus mechanism. Currently we apply Erdős-Rényi Model and Barabási-Albert model
- Agent/Node
Each agent is a miner, which has its own attributes (e.g., state, hash power) and behavior (e.g., gossip, mining).
- Block Tree
For simplicity and efficiency, blockchain is not stored individually, instead, all nodes share one global block tree.

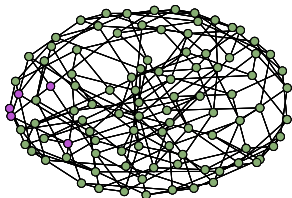
Notes

- In bitcoin network, there are nodes and miners. But currently in our model, we assume all nodes could mine. All nodes have hash power. But, we could easily apply pure node(no mining) by adjusting its hash power to zero.
- In general, the model is in block level. Has nothing to do with transactions. But, as gillespie algorithm is open for stacking new function, in theory, I think it is able be expended to transaction level.

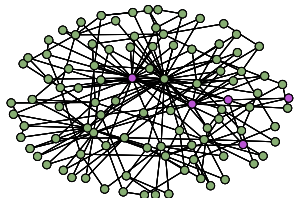
Model description (ii)

Network Topology

- To construct a common network, we apply 2 type of network topology: Erdős-Rényi Model and Barabási-Albert model.



Erdős-Rényi Model, Degree = 4



Barabási-Albert model, Degree = 2



Notes

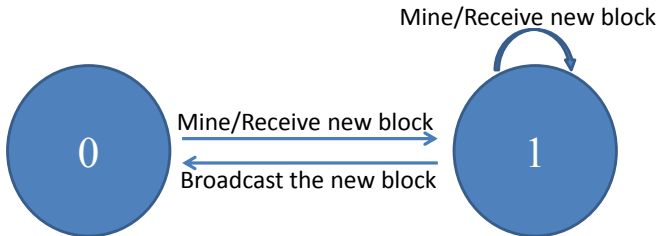
- The great advantage of this model is its flexibility. It could be easily tuned to any network topology.
- As for bitcoin network, a new node is usually bootstrapped by connecting to nodes in the seedlist, which makes the topology similar to Barabási-Albert model.

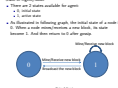
Model description (iii)

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State of Agent/Node

- There are 2 states available for agent:
 - 0, initial state
 - 1, active state
- As illustrated in following graph, the initial state of a node is 0. When a node mines/receives a new block, its state become 1. And then return to 0 after gossip.





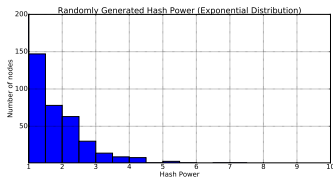
Notes

- The chart also shows a special case: if a node with active state mines/receives a new block, it will keep its state.

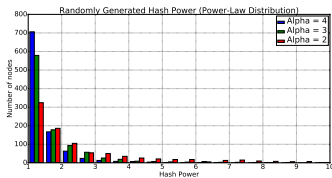
Model description (iii)

Hash Power of Agent

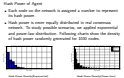
- Each node on the network is assigned a number to represent its hash power.
- Hash power is never equally distributed in real consensus network. To study possible scenarios, we applied exponential and power-law distribution. Following charts show the density of hash power randomly generated for 1000 nodes.



Hash Power Density(Exponential)



Hash Power Density(Power-Law)



Notes

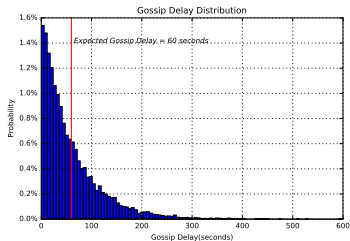
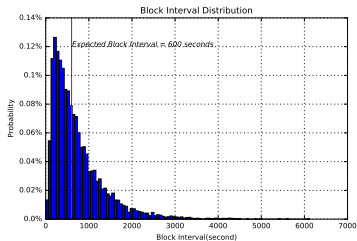
- Currently, the difficulty adjustment is not applied yet. So the 'hash power' of each miner is constant in each repetition.

Model description (iii)

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Mine / Gossip

- In general, each agent has 2 behaviors: mine and gossip.
- On system level, new block is generated according to given mining interval (600 seconds in case of bitcoin). On agent level, the expected gossip delay reflects time period for blocks propagating from one node to another.
- Both mining interval and gossip delay follow Poisson Distribution.





Notes

- the distribution chart for block interval is only for on-chain blocks.
- The peak of block interval distribution is larger than zero, because this is absolute interval time which includes also the gossip delay.
- I'm not sure whether net delay is clear for everyone, so I use gossip delay.
- The detail gossip process is, the node will exchange block information with its neighbor one by one, comparing the height of its new block H_{self} with its neighbors' (H_{nb}).
 - if $H_{self} > H_{nb}$, update neighbor's view of blockchain.
 - if $H_{self} < H_{nb}$, update its own view of blockchain.
 - if $H_{self} = H_{nb}$, continue gossiping with other neighbors.

Block Tree

- The root does not store block chain separately, instead, it stores its content built over global block tree.
- When the root stores a new block, the block will be attached over global block tree. By storing block of an block tree, each node would have its local copy of blockchain.

Photo: <https://www.khanacademy.org>

Notes

Summarize for Tuning Parameters

- Network Topology
- Average Network Degree
- Number of Nodes
- Hash Power Distribution
- Expected Value for Gossip Delay
- Expected Value for Block Interval

Summary for Testing Parameters

- Network Topology
- Average Network Degree
- Number of Nodes
- Node Power Distribution
- Expected Value for Group Degree
- Expected Value for Block Degree

Average Network Degree, just for Erdős-Rényi Network.

Performance Measurement

- Number of blocks
number of total blocks, number of orphaned blocks, number of valid blocks
- Branches
number of branches, the longest branches (with the most blocks)
- Propagation time
Propagation time measure the time one block needs to reach all nodes. Specifically, we measure average propagation time and max propagation time.

Performance Measurement

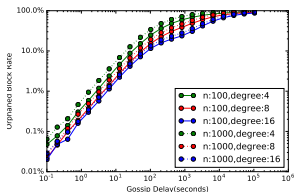
- Number of lines
number of total lines, number of syntactically correct
or valid lines
- Branches
number of branches, the largest branch(es) with the most
lines
- Propagation time
Propagation time measure the time one block needs to reach
all nodes. Specifically, we measure average propagation time
and max-propagation time.

Notes on what you meant

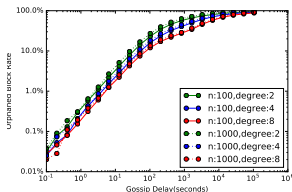
Results (i): Number of Orphaned Blocks

16

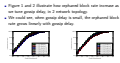
- Figure 1 and 2 illustrate how orphaned block rate increase as we tune gossip delay, in 2 network topology.
- We could see, when gossip delay is small, the orphaned block rate grows linearly with gossip delay.



F1.Erdős-Rényi/Exponential.



F2.Barabási-Albert/Exponential.



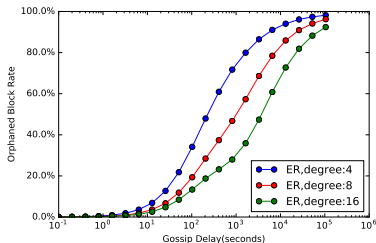
Notes

- All test is based on 100-day simulation, or around 14400 blocks. In both chart, the number of orphaned blocks move slowly to the ceiling value in the end.
- some simple observations, e.g., number of orphaned block increase as network size larger/degree smaller
- As the simulation day is fixed, the total block number is constant. Number of orphaned blocks and orphaned block rate behave the same.
- If the explanation for 'plateau effect' make sense, it could apply to the overall chart. Thus, the line in the chart should more or less mirror the cumulative distribution of block interval.

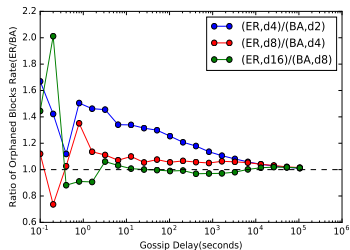
Results (i): Number of Orphaned Blocks

17

- In this chart, we put the results from 2 topology together. To give a clearer view, the chart shows only for network with 1000 nodes. We could clearly observe that, with the small degree, BA model generates less orphaned blocks than ER model. But BA's advantage disappears as degree increases.



F3. Erdős-Rényi, Exponential, 1000 nodes.

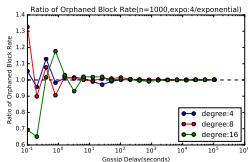
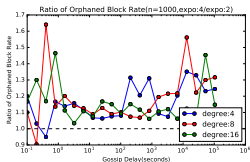
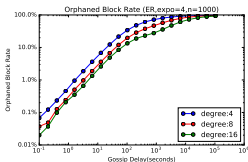


F4. Orphaned Block Rate Ratio (ER/BA).

Results (i): Orphaned Block Rate (Power law)

18

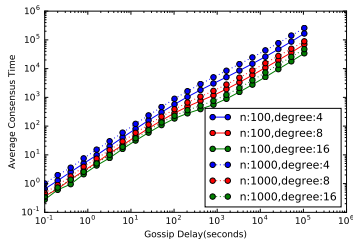
- First chart shows orphaned block rate for scenario hashrate follow powerlaw distribution. $\text{expo}=4, 1000$ nodes
- Second chart take ratio of orphaned block rate, $\text{expo}=4/\text{expo}=2$. It shows orphaned block rate tends to be higher with higher expo .
- Last chart take ratio of orphaned block rate, $\text{expo}=4/\text{exponential}$. It shows Powerlaw($\text{expo}=4$) and exponential perform the same.



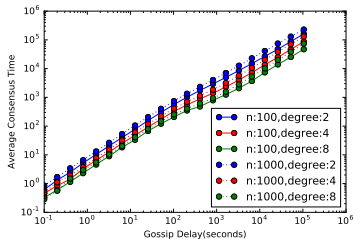
Results (ii): Consensus times

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- Average consensus time is the average time cost for one block to reach all nodes in the network.
- Average consensus time grows linearly with gossip delay in both topology.

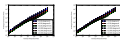


F4.Erdős-Rényi/Exponential.



F5.Barabási-Albert/Exponential.

- Average consensus time is the average time used for one block to reach all nodes in the network.
- Average consensus time grows linearly with group delay in both scenarios.

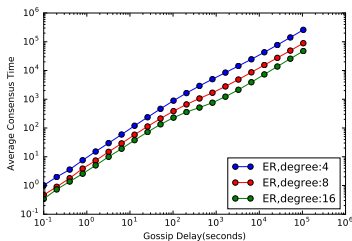


- the smaller the degree/the larger the network, the longer the avg consensus time.
- some details about propagation time
 - propagation time is only recorded for on-chain blocks.
 - In many cases(especially when net-delay is large), the block is more likely to free ride its descendant to reach all nodes. For example,
 - node one holds blockchain: b1-b2-b3-b5-b6-b7
 - node two holds blockchain: b1-b2-b4
 When node one gossips with node two, in the surface, it's block b7 who is propagating. But in fact, b3,b5 and b6 all benefit from b7's propagation. This explains why in video 3, we never see a color occupying the whole map, but consensus is achieved under map.

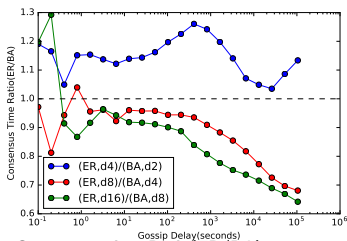
Results (ii): Consensus Time

20

- F6 shows the consensus time for 1000 nodes ER network
- F7 calculates the ratio of consensus time(ER/BA) in three different degree level. There is no clear stylized pattern.



F6. Erdős-Rényi, Exponential, 1000 nodes.

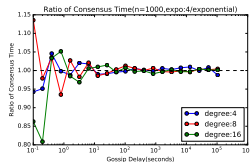
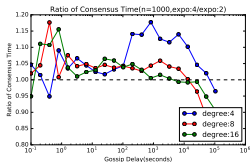
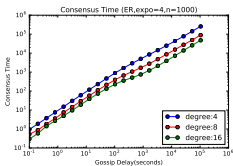


F7. Consensus Time Ratio(ER/BA).

Results (ii): Consensus Time(power law)

21

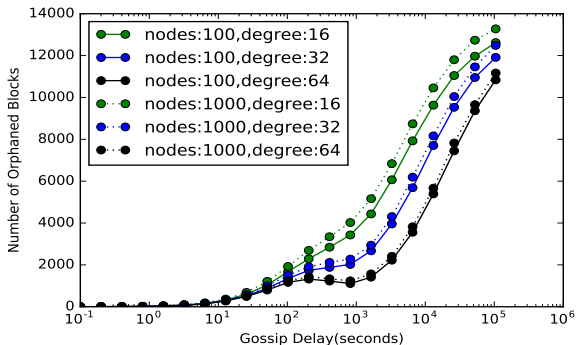
- First chart records the consensus time for scenario hashrate follow powerlaw distribution. $\text{expo}=4, 1000$ nodes
- chart 2 takes ratio of orphaned block rate, $\text{expo}=4/\text{expo}=2$. For most of the time, $\text{expo}=4$ takes longer consensus time
- Last chart take ratio of orphaned block rate, $\text{expo}=4/\text{exponential}$. It shows generally Powerlaw($\text{expo}=4$) and exponential perform the same.



Results (iii): Plateau Effect

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- Plateau effect is interesting evidence, which shows that as gossip delay increase, the number of orphaned block cease to increase for a while.



F7.Erdős-Rényi/Exponential.

Results (iii): Plateau Effect

A possible explanation

- There is no robust math yet to prove this. But we think this could be related to block generating rate.
- Detail explain, see notes.

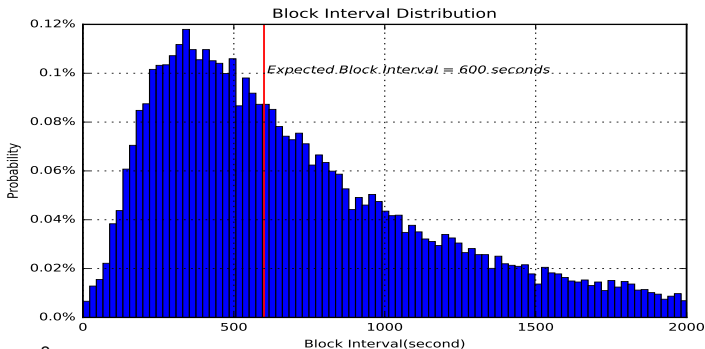


Figure 8



F8 is an amplified version of block interval distribution. We could understand how orphaned block generated by associating block generating time with consensus time. Not absolutely but it happens that, if consensus time is smaller than the generating time of next block, an orphaned block is avoided; if consensus time is larger than the generating time of next block, an orphaned block is possible to be generated. So, imagine if a given consensus time T cut the distribution in F8 into 2 parts, then its left side is the place where orphaned blocks born. Let's say the peak of F8 is 300 seconds. As consensus time changes from 0 to 300 seconds, the opportunity to generating orphaned block increase sharply. But as consensus time goes on increasing, the number of possible orphaned bloce still increases, but it slows down sharply. So, 300 seconds is the reverse point in this case. And the more effecient*(larger degree) a network is, the stronger it reacts to the reverse point.

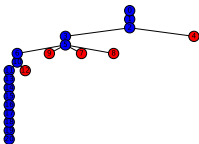
Results (iv): Network Evolution Demo

24

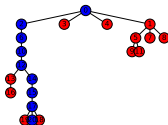
To understand the network evolution intuitively, here we illustrate the block tree and network evolution video in three different gossip delay scenarios.



gossip delay=1s(video)



gossip delay=100s(video)



gossip delay=1000s(video)

- By tuning different parameters in consensus mechanism, we get to know exactly how consensus performance is influenced by those parameters.
- We first compare the consensus performance based on 2 popular network topologies. It turns out that, in general, BA tends to produce less orphaned blocks than ER.
- Also, we examined the relationship between gossip delay and average consensus time.
- By studying 'plateau effect', we could understand better how block interval distribution and average consensus time could jointly influence the number of orphaned blocks.

- By using different parameters in consensus evaluation, we get an even clearer view consensus performance is influence by those parameters.
- We first compare the consensus performance based on 2 specific network topology. It turns out that, in general, 2D mesh is prefer the replicated blocks than 1D.
- Also, we measured the relationship between group delay and average consensus time.
- In the study of group delay, we could understand better how block size of distribution and average consensus time could jointly influence the number of replicated blocks.

notes

- What can we do with this?
Provide test data and basic knowledge for current blockchain improvement and future blockchain design.
The model is expected to simulate more consensus mechanism(e.g., proof of stake.), such we are able to provide performance measurement across platform.
- Why is this important
- Highly efficient, strong base for arbitrary questions

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